Topics to Cover: Linearity and Superposition Principle. Superposition of two collinear oscillations having (1) equal frequencies and (2) different frequencies (Beats). Superposition of N collinear Harmonic Oscillations with (1) equal phase differences and (2) equal frequency differences.

Already discussed in class: a) Linearity and Superposition Principle.

b) Superposition of two collinear oscillations having equal frequencies.

Recapitulation:

The superposition principle: The resultant of two or more harmonic displacements is simply the vector sum of the individual displacements."

Superposition of two collinear oscillations having equal frequencies (by analytical method):

When two SHMs of equal frequencies (ω) but having different amplitudes (A_1 and A_2) and differing by phase δ acts on a system in the *x*-direction and given by

$$x_1 = A_1 \sin \omega t \text{ and } x_2 = A_2 \sin(\omega t + \delta)$$

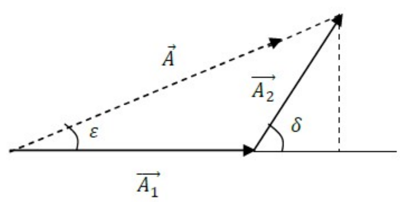
then the resultant motion will be a SHM with same frequency given by

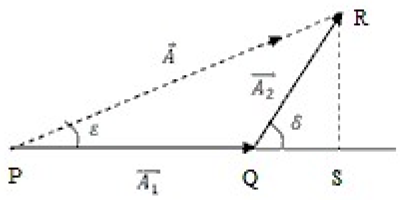
$$x = x_1 + x_2 = A \sin(\omega t + \varepsilon)$$

where, $A = \sqrt{A_1^2 + 2A_1A_2 \cos\delta + A_2^2}$ and $\tan\varepsilon = \frac{A_2 \sin}{A_1 + A_2 \cos}$;

In today's class: Superposition of two collinear oscillations having equal frequencies (by vector method):

We can arrive at the same results for the superposition of two collinear harmonic oscillations by using vector method also. We know SHM can be represented as rotating vector. We will make use of this representation to obtain the resultant of the superposition of two harmonic oscillations here. Let us represent the first SHM by a vector of magnitude A_1 and the second SHM by the vector of magnitude A_2. These two vectors are shown in the figure below. We assume here that the phase angle of the first vector is zero and for the second vector it is equal to the phase difference δ.





Let $|\overrightarrow{A_1}| = PQ$ and $|\overrightarrow{A_2}| = QR$ $RS \perp$ on extended part of PQ. Then $QS = QR\cos\delta$ and $RS = QR\sin\delta$. Hence magnitude of resultant vector $\vec{A}$, i.e.,

$$|\vec{A}| = PR = \sqrt{PS^2 + RS^2}$$

$$= \sqrt{(PQ + QS)^2 + RS^2}$$

$$= \sqrt{(PQ + QR\cos\delta)^2 + (QR\sin\delta)^2}$$

$$= \sqrt{(PQ)^2 + 2PQ.QR\cos\delta + \{(QR\cos\delta)^2 + (QR\sin\delta)^2\}}$$

$$= \sqrt{(PQ)^2 + 2PQ.QR\cos\delta + (QR)^2}$$

$$= \sqrt{(|\overrightarrow{A_1}|)^2 + 2|\overrightarrow{A_1}|.|\overrightarrow{A_2}|\cos\delta + (|\overrightarrow{A_2}|)^2} = \sqrt{A_1^2 + 2A_1A_2\cos\delta + A_2^2}$$

And,

$$\tan\varepsilon = \frac{height}{base} = \frac{RS}{PS} = \frac{RS}{PQ+QS} = \frac{QR\sin\delta}{PQ+QR\cos\delta}$$

$$= \frac{|\overrightarrow{A_2}|\sin\delta}{|\overrightarrow{A_1}|+|\overrightarrow{A_2}|\cos\delta}$$

$$= \frac{A_2\sin\delta}{A_1+A_2\cos\delta};$$

Thus we see expression for amplitude (*A*) and phase constant (ε) of resultant motion obtained in analytical and vector method is same.

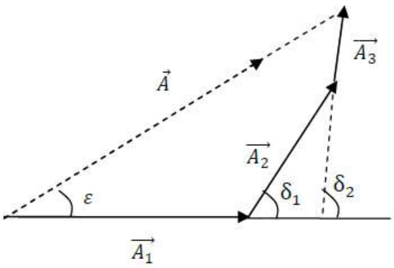
The vector method can easily be extended to more than two vectors. For example, if we have 3 vectors of same frequency, acting on a body along x-direction and given by

$$x_1 = A_1\sin\omega t$$

$$x_2 = A_2\sin(\omega t + \delta_1)$$

$$x_3 = A_2\sin(\omega t + \delta_2)$$

they can be represented as shown in the figure below.



The resultant vector is given by,

$$x = x_1 + x_2 + x_3 = A\sin(\omega t + \varepsilon)$$

Using trigonometry, we can prove that

$$A = \sqrt{(A_1 + A_2\cos\delta_1 + A_3\cos\delta_2)^2 + (A_2\sin\delta_1 + A_3\sin\delta_2)^2}$$

And

$$\tan\varepsilon = \frac{Height}{Base} = \frac{A_2\sin\delta_1 + A_3\sin\delta_2}{A_1 + A_2\cos\delta_1 + A_3\cos\delta_2}$$

Later on we shall apply this vector method to determine the resultant of superposition of N number of linear SHM having equal phase differences.

We shall end this class solving following numerical problems:

Problem 1: A particle is subjected to two simple harmonic oscillations

$$x_1 = A_1\sin\omega t$$

$$x_2 = A_2\sin\left(\omega t + \frac{\pi}{3}\right)$$

Determine (a) the displacement at t = 0, (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.

Problem 2: Calculate the amplitude and initial phase of the harmonic oscillations obtained by the superposition of two collinear oscillations represented by the following equations:

$$x_1 = (0.02)\sin\left(5\pi t + \frac{\pi}{2}\right) \text{ meter,}$$

$$x_2 = (0.03)\sin\left(5\pi t + \frac{\pi}{4}\right) \text{ meter}$$

Problem 3: A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

Next class: Superposition of two collinear oscillations having different frequencies (Beats).